Advanced Probability Modelling and the Prediction of Corporate Bankruptcy

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Abstract

Over the past three decades a sizeable international literature on financial distress prediction has developed (Altman, 2001). However, most empirical studies to date have relied on fairly simplistic modelling techniques, such as multiple discriminant analysis, binary logistic or rudimentary multinomial logit models (MNL). Recent research in discrete choice modelling has highlighted the limitations of these models, particularly in relation to their restrictive statistical assumptions and their failure to incorporate firm-specific observed and unobserved heterogeneity (both within and between firms) of any kind. Advanced probability models, such as mixed logit, not only have more appealing statistical properties but also allow for a high level of behavioural richness and definition to be specified in model estimation. This added flexibility and sophistication can improve the explanatory and predictive performance of financial distress models. We discuss the theoretical underpinnings of mixed logit and demonstrate its empirical usefulness in the context of a three-state failure model. Comparisons of model-fit statistics and out-of-sample forecasting accuracy indicate that the mixed logit model outperforms standard logit by significant margins.

Keywords

Bankruptcy Prediction Models
Mixed Logit
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Introduction

The prediction of firm financial distress has occupied the attention of accountants and financial economists for many decades now (Altman, 2001). Bankruptcies can impose a significant economic and social cost on the economy. As a result, the development of more accurate forecasting techniques is of importance to a variety of user groups who may have an existing or potential stake in business enterprises, including investors, creditors, managers, suppliers, employees and regulators. Distress forecasts are now used for many purposes, including monitoring of the solvency of financial and other institutions by regulators, assessment of loan security, going concern evaluations by auditors, the measurement of portfolio risk, and the pricing of defaultable bonds, credit derivatives and other securities exposed to credit risk (Scott, 1981; Shumway, 2001; Altman, 2001; Duffie and Singleton, 2003; Jones and Hensher, 2004).

While there is extensive interest in financial distress prediction among researchers and practitioners, much of the literature has relied on relatively simplistic multiple discriminant analysis (MDA), binary logistic or probit analysis or rudimentary multinomial logit models to predict corporate failure (MNL) (see e.g., Altman 1968; Altman, Haldeman, and Narayanan, 1977; Ohlson, 1980; Zmijewski, 1984; Lau, 1987).

The major limitation of the financial distress literature (and other related accounting literatures) is that there has been no recognition of the major developments in discrete choice modelling in recent years which has increasingly relaxed behaviourally questionable assumptions associated with the IID condition (independently and identically distributed errors) and allowed for observed and unobserved heterogeneity to be incorporated into model estimation. The latter in particular has been shown in other fields of the social sciences to have an important role to play in explanation and prediction (see Train, 2003).

A related problem is that most studies to date have modelled failure as a simplistic binary classification of failure or nonfailure (see Jones, 1987). The relevance of two state failure models has been questioned by many (particularly practitioners), one reason being that the strict legal concept of bankruptcy may be misleading, as not all corporate bankruptcy filings reflect the underlying economic reality of corporate financial distress (Lau, 1987; Delaney, 1991). Delaney (1991) for instance illustrates how some US firms have misused Chapter 11 bankruptcy protection provisions for their own economic gain. The two state model can also conflict with underlying theoretical models of financial failure and limits the generalisability of empirical results to other types of financial distress (other than outright failure) which can be observed in practice (Scott, 1981; Lau, 1987; Bahnson and Bartley, 1992; Hill et al., 1996). Other commentators have pointed out that the practical risk assessment decisions of banks and other lenders usually cannot be reduced to a simple pay off space of only two possible outcomes: failed and nonfailed (Ward, 1994; Ohlson, 1980).

The purpose of this paper[1] is to discuss the theoretical significance of the mixed logit model and provide an empirical illustration of its performance in the context of financial distress prediction. We introduce a three state failure model for the purposes of our empirical illustration. The remainder of this paper is organized as follows: section 2 discusses the theoretical foundation of the mixed logit model and how it improves on more basic discrete choice models used in much previous research and section 3 outlines the research methodology and empirical results. This is followed by concluding remarks.

Theoretical Foundations of the Mixed Logit Model

Mixed logit is the latest among a new breed of econometric models being developed out of discrete choice theory (Train, 2003).

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[1] This paper is based on the study by Jones and Hensher (2004) and takes an applied approach to the discussion of probability modelling.

Discrete choice theory is concerned with understanding the discrete behavioural responses of individuals to the actions of business, markets and government when faced with two or more possible outcomes (or choices) (Louviere et al., 2000). Discrete choice theory has developed from the microeconomic theory of consumer behaviour, such as the formal definition of agent preferences as inputs into a choice or outcome setting as determined by the utility maximization of agents. Given that the researcher has *incomplete* knowledge on the information inputs of the agents being studied, it is only possible to explain a choice outcome up to a *probability* of it occurring. This is the basis for the theory of random utility (see Louviere et al., 2000 for a review of literature). While random utility theory has developed from economic theories of consumer behaviour, it can be applied to any unit of analysis (such as firm failures, bond ratings or takeovers etc) where the dependent variable is *discrete*.

The concept of behavioural heterogeneity (individual variations in tastes and preferences), and how this impinges on the validity of various theoretical and empirical models has been the subject of much recent attention in the economics literature and elsewhere (see e.g., Grandmont, 1982; 1992; Kneip, 1999). However, econometric techniques to model heterogeneity have taken time to develop (particularly software algorithms), despite a long standing recognition that failure to do so can result in inferior model specification, spurious test results and invalid conclusions (Louviere, Hensher and Swait, 2000; Train, 2003).

Starting with the simple binary logit model, research progressed during the 1960s and 1970s to the multinomial logit (MNL) and nested logit models, the latter becoming the most popular of the generalized logit models. Although more advanced choice models such as mixed logit existed in conceptual and analytical form in the early 1970s, parameter estimation was seen as a practical barrier to their empirical usefulness. However, the development of simulation methods (such as simulated maximum likelihood estimation) enabled the open-form models such as mixed logit

to be estimated with relative ease (e.g., Stern, 1997).

Mixed logit and its variants have now developed very rapidly and have effectively supplanted simpler models in many areas of economics, marketing, management, transportation, health, housing, energy research and environmental science (Train, 2003). This can largely be explained in terms of the substantial improvements delivered by mixed logit over binary logistic and MNL models in terms of explanatory and predictive power.

Considering the case of firm failures, the main improvement is that mixed logit models include a number of *additional* parameters which capture observed and unobserved heterogeneity both within and between firms. In addition to fixed parameters, mixed logit models include estimates for the standard deviation of random parameters, the mean of random parameters and the heterogeneity in the means (these are interactions between contextual and/or continuous variables with random parameters).

The probability of failure for an individual firm using a binary logistic or MNL model is simply a weighted function of its fixed parameters (i.e., assumption of homogeneous preferences) with all other behavioural information assigned (incorrectly) to the error term. A fixed parameter essentially treats the standard deviation as zero (i.e no preference heterogeneity associate with a specific attribute) such that all the behavioural information is captured by the sample mean.

Standard logit models assume the population of firms is homogeneous across attributes (such as financial ratios) with respect to domain outcomes (i.e., levels of financial distress). For instance, the parameter for a financial ratio such as total debt to total equity is calculated from the sample of all firms (thus it is an average firm effect), and does not represent the parameter of an *individual* firm. The coefficient for total debt to total equity is the *same* for all firms. However, in reality we might expect the

coefficient for a failed firm such as (in Australia) One.Tel or HIH to be different to 'healthy' established firms such as BHP or Telstra. Mixed logit models take into account that individual coefficients for each firm can vary across the sampled population (hence the term 'random parameters').

For a mixed logit model, the probability of failure of a specific firm in a sample is therefore determined by the mean influence of each explanatory variable with a fixed parameter estimate within the sampled population, *plus*, for any random parameters, a parameter weight drawn from the distribution of individual firm parameters estimated across the sample. This weight is randomly allocated to each sampled firm unless there are specific rules for mapping individual firms to specific locations within the distribution of firm specific parameters.

It can be seen that the mixed logit model makes greater use of the behavioural information embedded in any dataset appropriate to the analysis. Ultimately, these conceptual advantages provide an improved foundation for explanation and prediction. The theoretical advantages of the mixed logit model are further considered in the formal specification and analysis of the model which now follows.

The Mixed Logit Model

Like any random utility model of the discrete choice family of models, we assume that a sampled firm (q=1,...,Q) faces a 'choice' amongst *I* alternatives in each of *T* occasions. Firms do not *choose* to fail per se, hence we use the phrase *outcome domain* (or simply outcome) as the descriptor of the observed *choice* outcome. A firm *q* is assumed to recognize the full set of alternative outcomes in occasion *t* and to focus on business strategies designed to result in the delivery of the outcome associated with the highest utility (i.e., nonfailure). The (relative) utility associated with each outcome *i* as evaluated by each firm *q* in occasion *t* is represented in a discrete outcome model by a utility expression of the general form:

$$U_{itq} = \beta_q X_{itq} + e_{itq} \qquad (1)$$

X_{itq} is a vector of explanatory variables that are observed by the analyst (from any source) and include attributes of the alternative outcomes (where observed), characteristics of the firm and descriptors of the decision context in occasion t, $ß_q$ and e_{itq} are not observed by the analyst and are treated as stochastic influences.

Jones and Hensher (2004) provide an intuitive explanation of how equation (1) operates in a practical outcome setting. To visualise this, think of the modelling task as being one of representing sources of variance that contribute to explaining a specific outcome such as firm financial distress. For a specific firm, equation (1) has variance potential associated with the coefficient attached to each observed characteristic (i.e., ß), to each observed characteristic itself (i.e., X) and the unobserved effects term (*e*). We could expand this equation out to reflect these sources of variance for three characteristics, defining '0' as observed and 'U' as unobserved, as (dropping the *q* and *t* subscripts):

$$U_i = (ß_{01}X_{01} + ß_{u1}X_{u1}) + (ß_{02}X_{02} + ß_{u2}X_{u2}) + (ß_{03}X_{03} + ß_{u3}X_{u3}) + e_i \qquad (1a)$$

It can be seen from equation (1a) that each characteristic is now represented by a set of observed and unobserved influences. In addition each parameter and characteristic can itself be expressed as some function of other influences, giving more depth in the explanation of sources of variance. As we expand the function out we reveal deeper parameters to identify. In the most restrictive (or simplistic) versions of the utility expression, we would gather all the unobserved sources together and replace (1a) with (1b):

$$U_i = ß_{01}X_{01} + ß_{02}X_{02} + ß_{03}X_{03} + (ß_{u1}X_{u1} + ß_{u2}X_{u2} + ß_{u3}X_{u3} + e_i) \qquad (1b)$$

and would collapse the unobserved influences into a single unknown by assuming that all unobserved effects cannot be related in any systematic way with the observed effects:

$$U_i = ß_{01}X_{01} + ß_{02}X_{02} + ß_{03}X_{03} + \boldsymbol{e_i} \qquad (1c)$$

Furthermore by defining a utility expression of the form in (1c) for each alternative outcome *i* and imposing a further assumption that the unobserved influences have the same distribution and are independent across alternatives, we can remove the subscript *i* attached to e. What we have is the functional form for the utility expressions of a multinomial logit (MNL) model. This intuitive discussion has highlighted the way in which an MNL model restricts, through assumption, the opportunity to reveal the fuller range of potential sources of influence on utility as resident throughout the full dimensionality of equation (1a). Explaining these fuller sources is equivalent to explaining the broader set of sources of observed and unobserved heterogeneity on an outcome domain.

A condition of the MNL model is that e_{itq} is independent and identically distributed (IID) extreme value type 1.[2] It is well documented in the literature that violation of the IID condition can lead to biased parameter estimates and misleading statistical inferences (Greene, 2005). To illustrate this a little further, consider the three failure states that we model in this study: nonfailure; insolvency; and outright failure. The IID condition implies that for any attribute (or combination of attributes), the error structure of the model (the deviation of each observation from the logistic regression equation) is identical across the distress outcomes, and further there is no correlation in the error structure between and within observations across the outcome alternatives. This assumption is likely to be very unrealistic, particularly

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[2] Extreme value type 1 (EV1) is a commonly used distribution in discrete choice analysis. The phrase 'extreme value' arises relative to the normal distribution. The essential difference between the EV1 and normal distributions is in the tails of the distribution where the extreme values reside. With a small choice set such as two alternatives this may make little difference because the resulting differences in the choice probabilities between the normal and EV1 is usually negligible. When there are more than two alternatives, however, there can be a number of very small choice probabilities. As a result, differences between the distributions can be quite noticeable (see Hensher, Rose and Greene, 2005).

when modelling company financial data which tends to exhibit a high degree of heteroscedasdicity (or unequal variance) under many circumstances. For example, operating cash flow is a variable that typically exhibits a higher degree of volatility (dispersion in distribution) in distressed and failed firms relative to 'healthy' nonfailed firms.

Violation of the IID condition can potentially be a serious problem in estimating discrete choice models (as it is in classical linear regression models). We would want to correct for this problem in some way in order to improve the performance and reliability of the model. The mixed logit model corrects for IID violations through a partitioning of the stochastic component into two additive (i.e., uncorrelated) parts. One part is correlated over alternative outcomes and heteroskedastic, and another part is IID over alternative outcomes and firms as shown in equation (2) (ignoring the t subscript for the present)

$$U_{iq} = \beta' x_{iq} + (\eta_{iq} + \varepsilon_{iq}) \qquad (2)$$

where η_{iq} is a random term with zero mean whose distribution over firms and alternative outcomes depends in general on underlying parameters and observed data relating to alternative outcome i and firm q; and ε_{iq} is a random term with zero mean that is IID over alternative outcomes and does not depend on underlying parameters or data.

The Mixed Logit class of models assumes a general distribution for η and an IID extreme value type 1 distribution for ε. That is, η can take on a number of distributional forms such as normal, Rayleigh, lognormal, and triangular. The probabilities in a mixed logit model do not exhibit the well known independence from irrelevant alternatives property (IIA).[3] This is handled through the

error components approach, which treats the unobserved information as a separate error component in the random component.

The standard deviation of a ß parameter accommodates the presence of preference heterogeneity in the sampled population of firms. While we could potentially handle this heterogeneity in the context of a fixed ß parameter (i.e., a basic logit model) through data segmentation (e.g., a different model for each industry segment) and/or attribute segmentation (e.g., separate ßs for different industry segments), in contrast to treating it all as random, the challenge of these (deterministic) segmentation strategies is in picking the right segmentation criteria and range cut-offs that account for statistically significant sources of preference heterogeneity. A random parameter representation of preference heterogeneity is more general; however such a specification also carries a challenge in that these parameters have a distribution that is unknown. Alternative analytical distributions can be evaluated such as the normal, lognormal and triangular (recent research suggests that the triangular distribution provides the best population-level predictive performance on a holdout sample – see Hensher and Jones, 2005).

Mixed logit models can be ordered or unordered. An ordered approach is appropriate where the dependent variable follows a natural ordinal ranking (e.g., a rating scale that ranges from low to high).

In this study we specify an ordered model[4], where the dependent variable represents a logical progression from the nonfailure state to outright failure.

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[3] IIA implies that the ratio of the probabilities of two alternatives is independent of the presence or absence of another alternative. Since violation is linked to $\varepsilon_{q,j,t,}$ then the challenge is to ensure that all of the influences are represented in the $\beta' x_{q,j,t}$ and /or the IID associated with the error component is relaxed.

[4] As explained in Jones and Hensher (2004), ordered models involve the specification of a single latent regression equation. The observation mechanism results from a complete censoring of the latent dependent variable. In an unordered model, a utility expression is specified for each outcome alternative. To identify an unordered model, one of the alternatives needs to be selected as the "base" category (which is set to zero). With an ordered specification of outcomes it is possible to allow a single characteristic or attribute to impact on all outcomes since the notion of an alternative is not applicable.

Explanatory variables can be readily incorporated into the mixed or random parameter logit model in the usual manner that they are incorporated into a regression equation. Estimates are obtained for both the parameters associated with each of the firm-specific variables, and the threshold parameters (which are a feature of *ordered* mixed logit models).

A Three-State Prediction Model

To illustrate the performance of the mixed logit model, we set out a three state model as follows:

State 0: non-failed firms;
State 1: insolvent firms. For the purposes of this study, insolvent firms are defined as: (i) failure to pay Australian Stock Exchange (ASX) annual listing fees as required by ASX Listing Rules; (ii) a capital raising *specifically* to generate sufficient working capital to finance continuing operations; (iii) loan default, (iv) a debt/total equity restructure due to a diminished capacity to make loan repayments.
State 2: firms who filed for bankruptcy followed by the appointment of liquidators, insolvency administrators or receivers.

We develop two samples for the purposes of model estimation and validation. The estimation sample is based on firm financial distress data collected between 1996 and 2000. Over this period we collect a sample of failed firms (state 2) and a sample of firms with solvency problems (state 1). We attempted to collect up to five annual reporting periods of data on all firms in categories 0,1 and 2, unless certain conditions described below were not met. To avoid the backcasting problem noted by Ohlson (1980), data are collected only from the financial statements already in the public domain on the date the failure is first made known to the market. The same procedure is followed for firms in state 1. To avoid over sampling problems and error rate biases associated with matched pair designs (see e.g., Casey and Bartczak,

1985; Gentry, Newbold and Whitford 1985; Jones 1987) we use a sample of failed and nonfailed firms which better approximates actual fail rates in practice (Zmijewski, 1984). This produces a final useable sample of 2,838 firm years in the nonfailed state 0; 78 firms years in state 1; and 116 firm years in state 2.

A validation sample is collected for the period 2001-2003 using the same definitions and procedures applied to the estimation sample. This produces a final useable sample of 4,980 firm years in the nonfailed state 0; and 119 and 110 firms years in states 1 and 2, respectively. The larger sample of nonfailed firms in the validation sample reflects a significant increase in the number of new listings on the ASX over this period and the fact that the more recent financial distress data was found to be relatively more complete than for the estimation sample.

Only publicly listed firms on the ASX are included in the estimation and validation samples. Furthermore, only firms who reported cash flow information under requirements of the Approved Australian Accounting Standard AASB 1026 "Statement of Cash Flows" are included in both samples. Compliance with AASB standards has been mandatory in Australia under the requirements of the existing Corporations Act (1991) of the day; hence we could find very few instances of noncompliance with AASB 1026. In a very small number of cases, firms are deleted from both samples because no financial statement records could be found. Following the approach of Ohlson (1980) no firm is deleted simply because it is newly or recently listed, and some firms in both our samples only had one or two years of financial reports. Consistent with Ohlson (1980), if a firm produced its annual report after the announcement of failure, then its published financial report of the previous reporting period is used. In the estimation sample, the average lead time between the date of the previous annual report and the announcement of failure was approximately 11.2 months (and 10.4 months for validation sample) which is broadly consistent with the lead time

reported in other studies (Altman, 1968; Ohlson, 1980).

With respect to the sample of insolvent firms, we employ the same data collection procedures used for failed firms. The financial report prior to the indication of the firm's solvency problem is used for estimation purposes. Whether a firm experienced a solvency problem (as defined in this study) is ascertained from the analysis of the ASX's Signal G releases.

For the estimation sample, financial statement data is collected on firms in each of the three states from four major sources: (i) Aspect Financial Pty Ltd's Financial Analysis Database (2003) and DatAnalysis Database (2003) – two leading Australian financial database sources which contains up to 15 years of historical data on all listed companies in Australia (ii) Huntley's Delisted Company Database (1993-1999), which contains all delisted firms in Australia up until 1999; (iii) ASX Market Comparative Analysis, 2003; and (iv) company financial statements collected from the Australian Securities and Investment Commission (ASIC). For the validation sample, all the financial data and failure statistics are generated from a customised data feed provided commercially to the authors by AspectHuntley Pty Ltd.

Explanatory Variables

We test a number of financial variables used in research over the last three decades (see e.g., Beaver, 1966; Altman, 1968; Altman, Haldeman and Narayan 1977; Ohlson, 1980; Zemjewski, 1984; Casey and Bartczak 1985; Gentry, Newbold and Whitford, 1985; Jones 1987). These financial measures include ratios based on cash position; operating cash flow (CFO); working capital; profitability and earnings performance; turnover; financial structure; and debt servicing capacity. Furthermore, we test reported CFO rather than estimates of CFO (Hribar and Collins, 2002). Ratio measures based on reported CFO are the net operating cash flow number extracted from company cash flow statements prepared under AASB 1026.

We now briefly comment on the contextual variables. In contrast to previous research, which has tended to be restricted to industrial or manufacturing firms (Jones, 1987), this study tests the predictive value of financial variables on four major sectors: the old economy sector; the new economy sector; the resources sector and the financial services sector. This classification approach has been adopted for a variety of reasons: (1) it recognizes that industry sectors are structurally different and have different financing, operating and investing characteristics that can undermine inter-sector comparability and generalisability (Ohlson, 1980). We attempt to capture sector-specific affects in our modelling in order to make determinations about the generalisability of our results as well as assess the predictive value of our model to specific sectors; (2) the classification approach gives explicit recognition to the economic characteristics of Australian industry, particularly the emerging importance of the New Economy sector in Australia over the past decade, which now amounts to more than 60% of the market capitalization of the ASX (ASX Market Comparative Analysis, 2003). Firms in the New Economy sector are classified according to the ASX industry classification guidelines, outlined in the ASX Market Comparative Analysis (2003). These are: (i) health and biotechnology; (ii) high technology; (iii) internet firms; and (iv) telecommunications. Furthermore, in Australia, the resources sector constitutes the largest and single most important export industry – nearly 30% of all listed firms in Australia are resource companies (see ASX Market Comparative Analysis, 2003).

Table One: Fixed and Random Parameter Estimates and Model-Fit Summary for Final Mixed Logit Models (100 Halton draws)

Variables[a]		Mixed Logit Parameter Estimates (t-values)	MNL Parameter Estimates[b] (t-values)
Fixed parameters:			
Constant		-2.85 (-39.2)	-2.6703 (-33.75)
Total debt to gross operating cash flow		0.00895 (4.08)	-.001654 (-6.7)
Working capital to total assets		-0.0119 (-10.765)	0.00186 (4.62)
Resources sector (1,0)		-0.5063 (-3.032)	Ns[c]
New economy sector (1,0)		0.725 (4.101)	0.40586 (3.12)
Finance sector (1,0)		ns	0.33873 (2.38)
Sales revenue to total assets		ns	-.00095 (-3.06)
Cash resources to total assets		-	0.002186 (4.98)
Net operating cash flow to total assets		-	-.00091 9-2.87)
Total debt to total equity		-	ns
Cash flow cover		-	0.00042 (3.8)
Random parameter means:			
Cash resources to total assets		-0.0608 (-7.65)	
Net operating cash flow to total assets		-0.0171 (-9.21)	
Total debt to total equity		0.0009 (2.68)	
Cash flow cover		-0.0051 (-7.93)	
Standard deviations of random parameters:			
Cash resources to total assets		0.0827(13.173)	
Net operating cash flow to total assets		0.0122 (6.104)	
Total debt to total equity		0.00509 (12.66)	
Cash flow cover		0.0048 (6.836)	
Heterogeneity in means:			
Total debt to total equity*New_Econ[d]		-0.0076 (-5.75)	
Cash flow cover*New_Econ		0.00819 (5.287)	
Threshold Parameters:			
μ (0 to 1)	Mu(0)	0	0
μ (1 to 2)	Mu(1)	1.2611 (10.1)	1.0398 (16.6)
Log-likelihood at zero		-2057.46	-2057.46
Log-likelihood at convergence		-776.17	-1971.95
Sample size		2838	2838

Table One (Continued)

Descriptive Statistics				
Variable	**Mean**	**Std.Dev.**	**Minimum**	**Maximum**
New economy sector	0.165	0.371	0.000	1.000
Finance sector	0.138	0.345	0.000	1.000
Resources sector	0.204	0.403	0.000	1.000
Net operating cash flow to total assets	-2.159	26.910	-404.760	256.640
Cash flow cover	7.218	81.220	-1014.986	1134.000
Cash resources to total assets	19.050	25.124	0.000	177.000
Total debt to gross operating cash flow	4.375	14.367	0.000	229.520
Total debt to total equity	56.836	152.620	0.000	698.870
Working capital to total assets	3.751	77.675	-850.000	323.000
Sales revenue to total assets	66.050	104.070	0.000	1734.600

[a] The full list of financial and contextual variables examined in the analysis included: net operating cash flow to total assets; net operating cash flow to sales revenue; cash flow cover (net operating cash flow to annual interest payments); total debt to gross operating cash flow (where gross operating cash flow equals total receipts from customers minus payments to suppliers); two annual periods of negative CFO, coded 1 = yes; 0 = no; three annual periods of negative CFO; coded 1 equal yes; 0 = no; cash resources (cash, deposits and marketable securities) to total assets; cash resources (cash, deposits and marketable securities) to current liabilities; current assets to current liabilities; working capital (current assets – current liabilities) to total assets; total debt to total equity; total liabilities to total equity; total debt to total assets; total liabilities to total assets; market value of equity to book value of debt; interest cover (reported EBIT to annual interest payments); reported EBIT to total assets; return on equity (net profit after tax to total equity); return on assets (net profit after tax to total assets); annual growth in sales revenue; sales revenue to total assets; retained earnings to total assets; natural log of total assets (a control variable for size); new economy sector (if a new economy sector firm coded 1, 0 otherwise); resources sector (if a resources sector firm coded 1, 0 otherwise); old economy sector (if an old economy sector firm coded 1, 0 otherwise); finance sector (if a finance sector firm coded 1, 0 otherwise).
[b] Note that all MNL parameters are fixed.
[c] ns = not significant
[d] Interaction of total debt to total equity and the new economy sector variable (coded 1 if a new economy firm, 0 otherwise).

The resources sector is classified by the ASX as: (i) gold companies; (ii) other metals and (iii) diversified resources. Financial services are defined by the ASX as banks and finance houses, insurance companies and investment and financial services companies.

Old economy firms are defined for the purpose of this study as all firms not being in the new economy, resources and financial services sectors;[5] and (3) our classification scheme is sufficiently broad to preserve a statistically

sufficient sample size for each major sector, while maintaining meaningful contextual information about the impact of industry sector in our analysis.

Results

Table One summarises the overall model system for both the mixed logit model and MNL. The mixed logit has delivered a very good overall goodness of fit. The log-likelihood has decreased from -2057 (assuming no information other than random shares) to -776. However, for the MNL, the

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[5] Industries include: Alcohol & Tobacco; Building Materials; Chemicals; Developers & Contractors; Diversified Industrials; Energy; Engineering; Food & Households; Infrastructure & Utilities; Media;

Miscellaneous Industrials; Paper & Packaging; Property Trusts; Retail; Tourism & Leisure; and Transport.

model fit is not as good. The MNL log-likelihood has only decreased from -2057 to -1972. Using a likelihood ratio test we can calculate the likelihood ratio as -2*(1972-776) = -2392 at 8 degrees of freedom. This is chi-square distributed and at any level of significance the mixed logit is a substantial improvement over MNL.

The mixed logit results in Table One indicate that some variables have a single fixed parameter whereas other variables (four of them - cash resources to total assets, net operating cash flow to total assets, total debt to total equity and cash flow cover) have up to three parameters representing their role. Importantly, the unobserved heterogeneity as represented by the standard deviation parameters is statistically significant for all four financial variables. Further, we find that for total debt to total equity and cash flow cover, the interaction with the new economy dummy variable has produced a contextual effect suggesting that membership of the new economy has a differential influence on the role of these variables to the failure outcome.

If the researcher was to estimate a simple multinomial or binary logit model, the opportunity to establish the role of the mean and variance influence of a particular variable (through the structure of its parameter space) would be denied. This is an important finding and recognition of the amount of information loss that is caused by rigid model specifications. The ability to capture important relationships through a random parameter specification has meant that such information has not been assigned (incorrectly) to the IID random component as exists for a standard logit model.

Predictive Performance of the Mixed Logit Model

The overall predictive performance of the mixed logit model and MNL can be investigated by deriving the predicted probabilities for each firm for each outcome on our validation sample. In deriving the probability outcomes for the mixed logit model we have to recognize that some explanatory variables are a composite function of a mean parameter, a distribution around the mean and decomposition of the mean by some contextual effect (in our case it is the new

economy effect). Each individual firm is 'located' in parameter space on the normal distribution for four financial variables (cash resources to total assets, net operating cash flow to total assets, total debt to total equity and cash flow cover). The precise formulation used to derive the contribution to relative utility of each outcome is:

Preference Distribution for cash resources to total assets
= -.06082089+.08277721*normal density

Preference Distribution for net operating cash flow to total assets
= -.01711009+.01229528* normal density

Preference Distribution for total debt to total equity
= .00090835-.00766415*new_econ+ .00509659* normal density

Preference Distribution for cash flow cover ratio
= -.00519331+.008199972*new_econ +.00519331*normal density

where normal densities have mean zero and unit standard deviation. For each individual we randomly draw a location on the distribution given the mean and standard deviation and derive their overall contribution to 'relative utility'. This is derived a repeated number of times and averaged per firm. We calculate the set of three outcome probabilities using the formula set:

P0 = Phi(-Xb)
P1 = Phi(Mu1-Xb) - P0
P2 = 1 - P1-P0

Implementing a sample enumeration strategy on our hold out sample (as suggested by Jones and Hensher, 2004), we can compare the predictive performance of mixed logit and MNL. Table Two compares the forecasting accuracy of both models on pooled data, and on data 1, 3 and 5 reporting periods prior to failure. The overall results indicate that mixed logit has substantially better predictive accuracy than MNL across the pooled results, and in all reporting periods prior to failure. It is noted that the MNL is particularly poor in classifying financial distressed firms. For instance, in predicting state '2' or outright

failure, the MNL model's best performance is only 5% accuracy based on the pooled observations and its best result is 6.4% accuracy three reporting periods prior to failure. The MNL performs better in predicting state 1 (insolvent firms), though the accuracy rate is only 24% on the pooled data with a peak accuracy rate of 29% five reporting periods from the public announcement of insolvency problems. In contrast, mixed logit predicts state '2' with 95% accuracy based on the pooled observations, and is 95% accurate up to three reporting periods prior to failure, with accuracy rates falling to 78% five reporting

periods from failure. The model performs very well on predicting nonfailures, and the overall accuracy for predicting state 1 is also impressive, with over 90% accuracy three and five reporting periods prior to the public announcement of insolvency problems. Mixed logit has an overall forecasting accuracy (in terms of predicting accurately across all distress states) of 99.16% on the pooled data, 98.73% from the last reporting period, 99.6% accuracy from the third reporting period, and 98.9% accuracy from the fifth reporting period.

Table Two: Forecasting Performance of Final Mixed Logit and Multinomial Logit Models across Distress States 0-2

Pooled Data (Reporting Periods 1-5)						
	Nonfailure (0)		**Insolvent (1)**		**Outright Failure (2)**	
Model	**Actual**	**Predicted**	**Actual**	**Predicted**	**Actual**	**Predicted**
Mixed	95.60%	95.90%	2.28%	1.84%	2.10%	2.20%
MNL	95.50%	99.30%	2.34%	0.58%	2.15%	0.11%

Last reporting period prior to failure						
	Nonfailure (0)		**Insolvent (1)**		**Outright Failure (2)**	
Model	**Actual**	**Predicted**	**Actual**	**Predicted**	**Actual**	**Predicted**
Mixed	95.70%	96.20%	2.37%	1.71%	1.91%	2.02%
MNL	95.70%	99.37%	2.37%	0.59%	1.91%	0.02%

Third reporting period prior to failure						
	Nonfailure (0)		**Insolvent (1)**		**Outright Failure (2)**	
Model	**Actual**	**Predicted**	**Actual**	**Predicted**	**Actual**	**Predicted**
Mixed	95.70%	95.90%	2.02%	1.97%	2.17%	2.04%
MNL	95.60%	99.27%	2.32%	0.59%	2.03%	0.13%

Fifth reporting period prior to failure						
	Nonfailure (0)		**Insolvent (1)**		**Outright Failure (2)**	
Model	**Actual**	**Predicted**	**Actual**	**Predicted**	**Actual**	**Predicted**
Mixed	96.20%	95.80%	1.93%	1.77%	1.84%	2.37%
MNL	96.20%	99.38%	1.93%	0.57%	1.84%	0.04%

Conclusions

Despite the proliferation in the financial distress literature over the past three decades, the modelling techniques used to explain and predict corporate distress are less developed than other fields of the social sciences. Much of the literature has relied on relatively primitive binary logistic models, and in a few cases a rudimentary MNL approach. Multiple discriminant analysis (MDA) has been another popular discrete choice technique used widely

in the literature. However, MDA is even more restrictive in its assumptions than a basic logit model as it imposes further constraints (multivariate normality) on the covariates and produces only point estimates for each parameter, whereas the standard logit model only imposes strong distribution assumptions on the error structure. The probit model has also been used in some distress studies, but is more limited than logit because of its critical reliance on normal distributions (see McFadden and Train, 2000).

The focus of recent developments in the discrete choice literature has been on improving the behavioural realism of discrete choice models by relaxing the rigid assumptions associated with IID error terms in a manner that is conceptually enriching, computationally tractable and practical. The parameterisation of measures which capture observed and unobserved heterogeneity in model estimation is another important (and related) development. The mixed logit model is one approach that allows the analyst to relax the very rigid assumptions associated with IID, and allows a meaningful interpretation of the role of the mean and variance influence of a particular variable on an outcome domain. Such refinements hold much promise in this field of research. The results of this study confirm the superiority of mixed logit over standard approaches such as MNL. After adjusting for the number of parameters, mixed logit produced a substantially improved model-fit compared with MNL. Furthermore, the out-of-sample forecasting accuracy of the mixed logit design was much superior to

multinomial logit. Future research could test the performance of the mixed logit model in other popular discrete choice settings (such as takeovers research, bond ratings and accounting method choices). Furthermore, the mixed logit model can be tested against other potentially powerful advanced choice models (not discussed in this paper) such as latent class MNL and generalized nested logit (see Jones and Hensher, 2005 for more details).

References

Altman, E., (1968), "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy", *Journal of Finance* 23: 589-609.

Altman, E., Haldeman, R. and Narayan, P. (1977), "ZETA Analysis: A New Model to Identify Bankruptcy Risk of Corporations. *Journal of Banking and Finance* June: 29-54.

Altman, E., (2001), *Bankruptcy, Credit Risk and High Yield Junk Bond,* New York, NY: Blackwell Publishers.

Australian Accounting Standards Board, (1992), *Approved Australian Accounting Standard AASB 1026: Statement of Cash Flows*, Melbourne: Australian Accounting Research Foundation.

Australian Corporations Act (2001), (http://www.asic.gov.au/asic/ASIC)

Australian Stock Exchange, (2003), *Listing Rules*, Sydney, (http://www.asx.com.au/about/l3/ListingRules)

Australian Stock Exchange, (2003), *Market Comparative Analysis*, Sydney, (http://www.asxtra.asx.com.au).

Bahnson, P. and Bartley, J., (1992), "The Sensitivity of Failure Prediction Models", *Advances in Accounting* 10: 255-278.

Beaver, W., (1966), "Financial Ratios as Predictors of Failure", *Empirical Research in Accounting: Selected Studies, Supplement to Journal of Accounting Research* 4.

Boersch-Supan, A. and Hajvassiliou, V., (1990), "Smooth Unbiased Multivariate Probability Simulators for Maximum Likelihood Estimation of Limited Dependent Variable Models", *Journal of Econometrics,* 58: 347-368.

Brownstone, D., Bunch, D. and Train, K., (2000), "Joint Mixed Logit Models of Stated and Revealed Preferences for Alternative Fuel Vehicles", *Transportation Research,* 34: 315-338.

Casey, C. and Bartczak, N., (1985), "Using Operating Cash Flow Data to Predict Financial Distress: Some Extensions", *Journal of Accounting Research 23*: 384-401.

Clark, K. and Ofek, E., (1994), "Mergers as a Means of Restructuring Distressed Firms: and Empirical Investigation", *Journal of Financial and Quantitative Analysis* 29: 541-565. T. F.

Cheng, C.S., Liu, C. S. and Schaefer, T.F. (1997), "The Value-Relevance of SFAS No. 95 Cash Flows from Operations as Assessed by Security Market Effects", *Accounting Horizons* 11: 1-15.

Crask, M. and Fox, R., (1987), "An Exploration of the Interval Properties of Three Commonly Used Marketing Research Scales: a Magnitude Estimation Approach", *Journal of the Market Research Society* 29: 317-339.

DeAngelo, H. and DeAngelo, L., (1990), "Dividend Policy and Financial Distress: An Empirical Investigation of Troubled NSYE Firms", *Journal of Finance.* 45: 1415-1431.

Delaney, K., (1991), *Strategic Bankruptcy: How Corporations and Creditors Use Chapter 11 to their Advantage*, Berkeley: University of California Press

Duffie, D., and Singleton, K., (2003), *Credit Risk: Pricing, Measurement and Management,* Princeton: Princeton University Press.

Efron, B., (1975), "The Efficiency of Logistic Regression Compared to Normal Discriminant Analysis", *Journal of the American Statistical Association* 70: 892-898.

Elliot, J. and Kennedy, D., (1988), "Estimation and Prediction of Categorical Models in Accounting Research, *Journal of Accounting Literature* 7:202-242.

Financial Accounting Standards Board, (1987), *Statement of Financial Accounting Standards 95, Statement of Cash Flows,* Stamford, CT: FASB.

Gentry, J., Newbold, P. and Whitford, D., (1985), "Classifying Bankrupt Firms with Funds Flow Components", *Journal of Accounting Research* 23: 146-160.

Geweke, J., Keane, M. and Runkle, D., (1994), "Alternative Computational Approaches to Inference in the Multinomial Probit Model", *Review of Economics and Statistics* 76: 609-632.

Grandmont, J., (1987), "Distribution of Preferences and the `Law of Demand", *Econometrica.* 55:155-161.

Grandmont, J., (1992), "Transformation of the Commodity Space, Behavioural Heterogeneity, and the Aggregation Problem", *Journal of Economic Theory.* 75: 1-35.

Greene, W.H. (2003), *Econometric Software (2003): Limdep 8 Econometric Modeling Guide Volumes 2, Chapter 19,* Econometric Software Inc. New York and Sydney.

Henry, F., (1982), "Multivariate Analysis and Ordinal Data", *American Sociological Review* 47:299-304.

Hensher, D., (1984), "Achieving Representativeness of the Observable Component of the Indirect Utility Function in Logic Choice Models: An Empirical Revelation", *Journal of Business* 57:265-86.

Hensher, D.A., Louviere, J. and Swait, J., (1999), "Combining Sources of Preference Data", *Journal of Econometrics* 89:197-221.

Hensher, D.A. and Greene, W., (2003), "The Mixed Logit Model: The State of Practice", *Transportation*. 30:133-176.

Hensher, D.A., Rose, D. and Greene, W., (2005), *Applied Choice Analysis: A Primer*, Cambridge: Cambridge University Press.

Hensher, D.A. and Jones, S., (2005), "Forecasting Corporate Bankruptcy: Optimizing the Performance of the Mixed Logit Model", Working Paper, The University of Sydney.

Hill, N., Perry, S. and Andes, S., (1996), "Evaluating Firms in Financial Distress: An Event History Analysis", *Journal of Applied Business Research* 12:60-71.

Horowitz, J. and Louviere, J. (1993), "Testing Predicted Choices Against Observations in Probabilistic Discrete-Choice Models", *Marketing Science* 12(3):270-279.

Hribar, P. and Collins, D., (2002), "Errors in Estimating Accruals: Implications for Empirical Research", *Journal of Accounting Research* 40:105-134.

Ireland, J.C., (2003), "An Empirical Investigation of Determinants of Audit Reports in the UK", *Journal of Business Finance & Accounting* 30: 975-1016.

Jones, F., (1987), "Current Techniques in Bankruptcy Prediction", *Journal of Accounting Literature* 6:131-164.

Jones, S. and Hensher, D.A. (2004), "Predicting Firm Financial Distress: A Mixed Logit Model", *The Accounting Review* 79: 1011-1039.

Jones, S. and Hensher, D.A., (2005), "Towards an Improved Framework for Predicting Corporate Failure: A Comparison of Open vs. Closed Form Discrete Choice Models", Working Paper, The University of Sydney.

Jones, S. and Hensher, D.A., (2005a), "A Comparison of Open vs. Closed Form Discrete Choice Models", Working Paper, The University of Sydney.

Joy, M., and Tollefson, J., (1975), "On the Financial Applications of Discriminant Analysis", *Journal of Financial and Quantative Analysis* December: 723-739.

Kennedy, D., (1992), "Classification Techniques in Accounting Research: Empirical Evidence of Comparative Performance", *Contemporary Accounting Research* 8: 419-442.

Kneip, A., (1999), "Behavioral Heterogeneity of Structured Properties of Aggregate Demand", *Journal of Mathematical Economics* 31: 49-74.

Lancaster, K., (1966), "A New Approach to Consumer Theory", *Journal of Political Economy* 74: 132-157.

Lancaster, K., (1971), *Consumer Demand: A New Approach*, New York, NY: Columbia University Press.

Louviere, J., Hensher, D. and Swait, J., (2000), *Stated Choice Methods and Analysis* Cambridge: Cambridge University Press.

Lau, A.H., (1987), "A Five-State Financial Distress Prediction Model", *Journal of Accounting Research* 25: 127-138.

Leclere, M., (1999), "The Interpretation of Coefficients in N-Chotomous Qualitative Response Models", *Contemporary Accounting Research* 16: 711-747.

Leclere, M., (2000), "The Occurrence and Timing of Events: Survival Analysis Applied to the Study of Financial Distress", *Journal of Accounting Literature* 19: 158-189.

Libby, R. and Lipe, M., (1992), "Incentives, Effort, and the Cognitive Process Involved in Accounting Related Judgements", *Journal of Accounting Research* 30: 249-273.

Maddala, G.S., (1991), "A Perspective on the Use of Limited-Dependent and Qualitative Variables Models in Accounting Research", *The Accounting Review* 66: 788-807.

McFadden, D., (1976), "A Comment on Discriminant Analysis 'Versus' Logit", *Annals of Economic and Social Measurement* 5/4: 511-523.

McFadden, D. and Ruud, P., (1994), "Estimation by Simulation", *Review of Economics and Statistics* 76: 591-608.

McFadden, D. and Train, K., (2000), "Mixed MNL Models for Discrete Response", *Journal of Applied Econometrics* 15: 447-470.

Neill, J. D., Schaefer, T.F., Bahnson P.R. and Bradbury, M.E., (1991), "The Usefulness of Cash Flow Data: a Review and Synthesis", *Journal of Accounting Literature* 10: 117-150.

Ohlson, J., (1980), "Financial Ratios and the Probabilistic Prediction of Bankruptcy", *Journal of Accounting Research* 18: 109-131.

Powell, R., (1997), "Modelling Takeover Likelihood", *Journal of Business Finance and Accounting*. 24: 1009-1031.

Revelt, D. and Train, K., (2000), "Customer Specific Taste Parameters and the Mixed Logit: Households' Choice of Electricity Supplier", *Working Paper, Department of Economics, University of California, Berkely*.

Rosen, S., (1974), "Hedonic Prices and Implicit Markets: Product Differentiation in Competition", *Journal of Political Economy* 82: 34-55.

Sándor, Z., and Wedel, M. (2002), "Profile Construction in Experimental Choice Designs for Mixed Logit Models", *Marketing Science* 21: 455-475.

Scott, J., (1981), "The Probability of Bankruptcy: a Comparison of Empirical Predictions and Theoretical Models",

Journal of Banking and Finance 5: 317-344.

Shumway, T., (2001), "Forecasting Bankruptcy More Accurately: A Simple Hazard Model", *Journal of Business* 74: 101-124.

Stern, S., (1997), "Simulation-Based Estimation", *Journal of Economic Literature* 35: 2006-2039.

Train, K., (2003), *Discrete Choice Methods with Simulation.* Cambridge: Cambridge University Press

Ward, T., (1994), "An Empirical Study on the Incremental Predictive Ability of Beaver's Naïve Operative Flow Measure Using Four-State Ordinal Models of Financial Distress", *Journal of Business Finance & Accounting* 21: 547-562.

Winship, C. and Mare, R,. (1984), "Regression Models with Ordinal Variables", *American Sociological Review* 49: 512-525.

Zmijewski, M., (1984), "Methodological Issues Related to the Estimation of Financial Distress Prediction Models", *Journal of Accounting Research 22* (3): Supplement, 59-82.